

1. Function  $R$  is defined by  $R(x) = \frac{2x^2 - 18}{(5x - 15) \cdot (2 - x)}$ .
- Explain why 3 is not in the domain of this function.
  - Factor and simplify the expression for  $R(x)$ .
  - Show work which computes  $\lim_{x \rightarrow 3} R(x)$  or shows that limit does not exist.
  - What do the results of parts (b) & (c) imply about the graph of  $y = R(x)$  for  $x$  in  $[2, 4]$ ?

2. Consider the following statement.

$$\text{If } \lim_{x \rightarrow 5} f(x) \text{ exists and if } f(5) \neq 0, \text{ then } \lim_{x \rightarrow 5} \frac{1}{f(x)} \text{ exists.}$$

Explain why that general statement is not correct. [Hint: Is there a function for which that statement is false?]

3.
  - Explain why  $\lim_{x \rightarrow \infty} \sin(x)$  does not exist.
  - Show  $\lim_{x \rightarrow \infty} \frac{\sin(x)}{x}$  does exist and compute its value.

4.  $f(x) = \frac{x^2 + 2x - 3}{x^3 - 1}$  is not defined for  $x = 1$ . Is there a number  $c$  such that letting  $f(1) = c$  would yield a revised function which would be continuous at  $x = 1$ ? Explain your answer.

5. Find a value of parameter  $A$  for which the following function is continuous at 5. Explain why your choice of  $A$  “works”.

$$f(x) = \begin{cases} A + x & \text{if } x \leq 5 \\ A \cdot x & \text{if } x > 5 \end{cases}$$

6.
  - Use a **limit** to define the derivative function of  $f(x) = x^2 - 3$ .
  - Use the previous **limit** to verify that  $f'(2) = 4$ .

7. Let  $g(x) = \sqrt{x+4}$ . Use the limit definition of a derivative to verify that  $g'(0) = \frac{1}{4}$ .

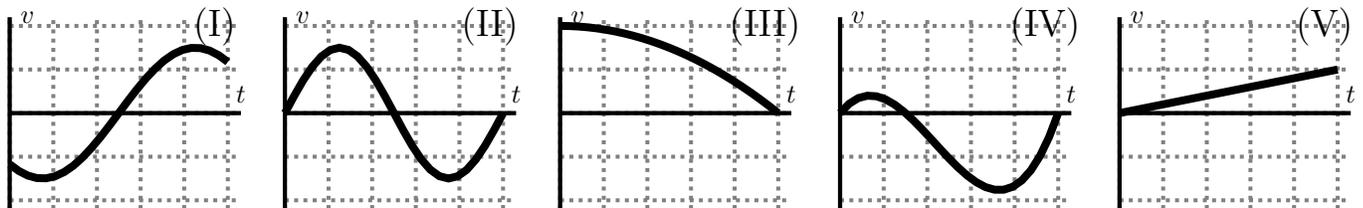
8. The line with equation  $3x - 4y = 7$  is tangent to the graph of  $y = f(x)$  at the point with  $x = -5$ .

  - $f(-5) =$
  - $f'(-5) =$

9. Use an appropriate local linear approximation to estimate the value of  $\sqrt[3]{7.98}$

10. An arrow is shot upwards. If the arrow hits the ground 20 seconds later, what was its initial velocity?

11. These graphs present velocity,  $v(t)$ , for several particles moving along the  $x$ -axis for times  $0 \leq t \leq 5$ . [All figures use the same scales.]



For each of the following description of a motion, identify which graph(s) show that behavior.

- has a constant acceleration
- ends up farthest to the left of where it started
- ends up farthest to the right from its starting point
- experiences the greatest initial acceleration
- has the greatest average velocity
- has the greatest average acceleration

12. Suppose  $f$  and  $g$  are differentiable functions with values shown in these tables.

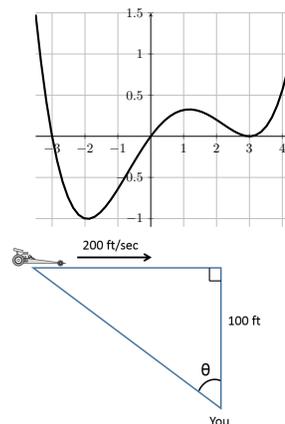
$x$	$f(x)$	$f'(x)$
2	3	5
4	-7	6

$x$	$g(x)$	$g'(x)$
2	4	-2
4	3	9

For each of the following parts, show how to use those table values to compute the specified derivative OR explain what extra information would be needed to finish that computation.

- a) If  $A(x) = f(x) - g(x)$ , then  $A'(4) =$   
 b) If  $B(x) = f(x) \cdot g(x)$ , then  $B'(2) =$   
 c) If  $C(x) = \frac{g(x)}{f(x)}$ , then  $C'(4) =$   
 d) If  $D(x) = f(g(x))$ , then  $D'(2) =$
13. Domain of function  $f(x) = \ln(|3x|)$  includes all non-zero real numbers. Show work which finds an expression for  $f'(x)$ .  
**Hint:** treat the case  $x > 0$  first, then consider the case  $x < 0$  separately. Simplify each result, then simplify further.
14. Differentiate some expressions involving Sine and Cosine several times.  
 a) Find the third derivative of  $f(x) = \sin(7 + x)$ .  
 b) Find the fifth derivative of  $g(x) = \cos(2x)$ .  
 c) Explain why repeatedly differentiating  $\sin(x)$  will never produce  $-2 \cos(x)$ .
15. Consider the equation  $e^y - 2x^3y + 4 = 5x$ .  
 a) Find an expression for  $\frac{dy}{dx}$ .  
 b) What condition must  $x$  and  $y$  satisfy so that the corresponding tangent line is horizontal?  
 c) What condition must  $x$  and  $y$  satisfy so that the corresponding tangent line is vertical?
16. Suppose  $g$  is a differentiable function such that  $-1 \leq g'(x) \leq 3$  for all  $x$  in  $[0, 12]$ . Also suppose  $g(2) = 5$ .  
 a) What is the smallest possible value for  $g(7)$ ? Explain your reasoning.  
 b) What is the largest possible value for  $g(7)$ ? Explain your reasoning.

17. Examine this graph of the derivative of some function  $f$ .  
 a) Determine where  $f$  is decreasing.  
 b) Locate the  $x$  values for the local extrema of  $f$ .  
 c) Determine where  $f$  is concave up.  
 d) Locate the  $x$  values for any inflection point of  $f$ .



18. Find the global extrema of the function  $f(x) = 2x - e^x$  on the interval  $[-1, 2]$
19. Examine this figure. You are taking a video of a racecar from a position that is 100 feet from the racetrack. The racecar is moving at constant rate of 200 feet-per-second. How fast is your viewing angle,  $\theta$  in this figure, changing at the moment when the car is directly in front of you?
20. Starting at time  $t = 0$  hours, water leaks out of a tank at rate  $r(t)$ , measured in gallons-per-hour. This table has some values for  $r(t)$ .

$t$ [hr]	0.0	0.5	1.0	1.5	2.0	2.5	3.0
$r(t)$ [gal/hr]	0.0	6	11	15	18	20	21

- a) What is the meaning of the quantity  $\int_1^2 r(t) dt$ ? Write your answer in terms relevant to this situation AND make it understandable by someone who does not know any calculus; mention any units that are appropriate.  
 b) Write a three-term Riemann sum to estimate  $\int_0^3 r(t) dt$  [this is not the definite integral given in part (a)]. Give your answer as an expression in terms of numbers, but you do not need to do the arithmetic.
21. Evaluate the following definite integrals and antiderivative.  
 A)  $\int_{-1}^3 (w + 1) \cdot (w - 1) \cdot (w - 3) dw$       B)  $\int_1^{e^3} \frac{5}{x} - 6x^2 + \sqrt{x} dx$       C)  $\int 5e^t + \sin(t) dt$
22. Shade a region whose area is computed by  $\int_{-5}^5 \sqrt{25 - x^2} dx$ ; then use geometry to evaluate the definite integral.