This collection of problems shows the range of ideas you may encounter on our first test (Friday, 17–February). The actual test will provide space for you to show your work and interpret your results. Mere answers *with no work shown* are not eligible for partial credit. Points-per-problem are shown in brackets after the problem number, e.g., problem 1 is worth 16 points.

NOTE: **You may use a graphing calculator only during the last 15 minutes of the test.**

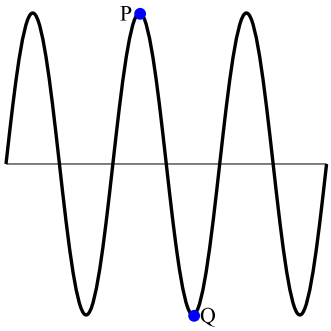
1)[16] Suppose $p(x) = x \cdot (5 - 3\,x)$ for all real values of x.

a)[6] State a formal definition (involving an appropriate limit) for $p'(2)$.

b)[10] Show work that finds the exact value of $p'(2)$ by evaluating the limit you wrote for part (a). *Note: you may check your numerical result in a variety of ways, but the necessary work for part (b) must begin with an expression involving an appropriate limit and must show key steps which reveal the exact value of that limit.*

2)[10] Let f be the sine function. Suppose points P and Q are consecutive highest and lowest points on the graph of f. [The figure has minimal detail, but may help your analysis.]

Compute the average-rate-of-change for function $f = \sin$ on the interval from P to Q.



3)[10] Find a value for A so that $f(x) = \begin{cases} A\cos(x) & \text{if } x < 0 \\ e^x - A & \text{if } x \geq 0 \end{cases}$ is continuous. Explain why your answer is correct.

4)[15] Classify each of the following statements as **TRUE** or **FALSE**, then discuss each classification. If a statement is false, then either explain why it is false or revise it to be correct; if a statement is true, then provide supporting evidence. Note: 60% of the credit on each part of this problem is for an adequate discussion.

a)[5] If a car's speed is 40 miles-per-hour at 2pm and 60 miles-per-hour at 3pm, then the distance traveled by the car between 2pm and 3pm must be between 40 and 60 miles.

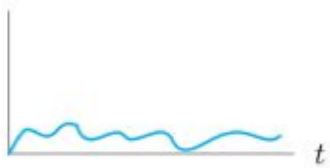
b)[5] The derivative of a function f at $x = a$ is the tangent line to the graph of f at $x = a$.

c)[5] Suppose $f(t)$ is the quantity [in kilograms] of a chemical produced after t minutes. Also suppose $g(t)$ is the quantity [in grams] of that chemical produced after t seconds. That implies $f'(t) = 60 \cdot g'(t)$.

5)[12] A vehicle moving along a straight road has distance $g(t)$ from its starting point at time t. Each of the following figures is the graph of $g'(t)$ for some context. (Assume scales on the vertical axes are all the same.)

P) 

Q) 

R) 

Match each graph with one of the following scenarios (2 points for each match), then sketch plausible graphs of both g and g' for the remaining scenario (6 points for your pair of sketches).

a) A bus drives on a popular route with no traffic.

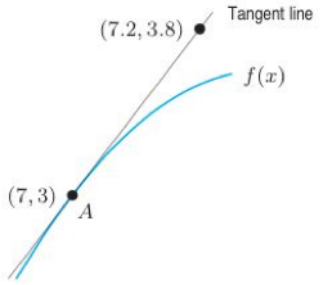
b) A taxi drives in heavy traffic conditions.

c) An ambulance responds to a medical emergency and then transports a patient to a hospital.

d) A car drives with no traffic and all green lights.

6)[10] Suppose $F(x) = \dfrac{3\,x + 7x^2}{4x^3 - 5\;x}$. Compute $\lim\limits_{x\to\infty} F(x)$ or explain why that limit does not exist.

7)[12] The figure shows graph of function f, the line tangent to graph of f at point $A = (7, 3)$, and point $(7.2, 3.8)$ on that tangent line. Use this information to complete the following statements.



a)[2] $f(\_\_\_\_\_) = \_\_\_\_\_$

b)[6] $f'(\_\_\_\_\_) = \_\_\_\_\_$

c)[4] $f(7.05) \approx f(7) + \_\_\_\_\_$

8)[15] In May 2007, the www.census.gov website summarized population changes by reporting there was one birth in the US every eight seconds, one death in the US every thirteen seconds, and one new international migrant into the US every twenty-seven seconds.

a)[10] Let $f(t)$ be the population of the US, where t is time in seconds measured from the start of May 2007. Find $f'(0)$. Identify the units for your answer.

b)[5] To the nearest second, how long did it take for the US population to add one person in May 2007?