Test 3 will be 21–Apr–2017; its tasks will emphasize ideas in chapters 4 and 5. Most of these problems were selected from previous versions of Test 3.

1. [20] Let $f(x) = x \cdot (x-1)^3$. Its first and second derivatives are

$$f'(x) = (4x-1)\cdot(x-1)^2 \qquad \text{and} \qquad f''(x) = 6\cdot(2x-1)\cdot(x-1)$$

For each part, give a **brief** explanation. Write NONE for any item where that is appropriate.

a)[2] Find all critical points of f. ___________________

b)[6] Locate where f is increasing and where it is decreasing.

- f is increasing on ___________________
- f is decreasing on ___________________

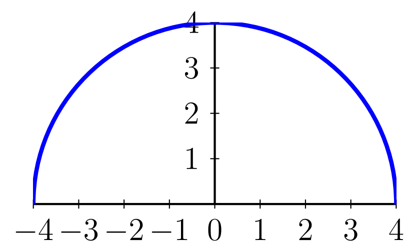
c)[6] Locate where, if anywhere, f attains a local extreme.

- f has a local minimum at point(s) with x-coordinate(s)
- f has a local maximum at point(s) with x-coordinate(s)

d)[6] Investigate concavity of the graph of f.

- Graph of f is concave down on ___________________
- Graph of f is concave up on ___________________
- Graph of f has inflection point(s) with x-coordinate(s) ___________________

2. [15] Upper-half of the curve with equation $x^2+y^2 = 16$ is shown. Many rectangles fit inside that curve. Consider just those rectangles with one side on the x-axis and the opposite side with its endpoints on the curve. Demonstrate use of calculus ideas to find dimensions of such a rectangle with maximal area.



3. [15] Suppose a point moves in the plane on the parametric curve

$$x(t) = 1+\cos(t) \qquad \text{and} \qquad y(t) = t+\sin(2t) \qquad \text{with} \qquad -0.5 \le t \le 3.5$$

a)[3] Show this particle never comes to a stop.

b)[6] For $-0.5 \le t \le 3.5$, is the particle ever moving straight up or straight down? If so, identify the t-value(s), the point(s) $\big(x(t), y(t)\big)$, and identify the direction (up or down).

c)[6] For $-0.5 \le t \le 3.5$, is the particle ever moving straight horizontally, left or right? If so, identify the t-value(s), the point(s) $\big(x(t), y(t)\big)$, and identify the direction (right or left).

4. [10] Show work which computes the following limits.

a) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x}}{\ln(x)} =$

b) $\displaystyle\lim_{x\to 0} \frac{e^{-x}-1+x}{x^2} =$

5. [15] An object moves along a straight line. At time $t = 0$, its velocity is $2\,\frac{\text{ft}}{\text{s}}$. For the next fifteen seconds, the object accelerates at the rate of $3\,\frac{\text{ft/s}}{\text{s}}$.

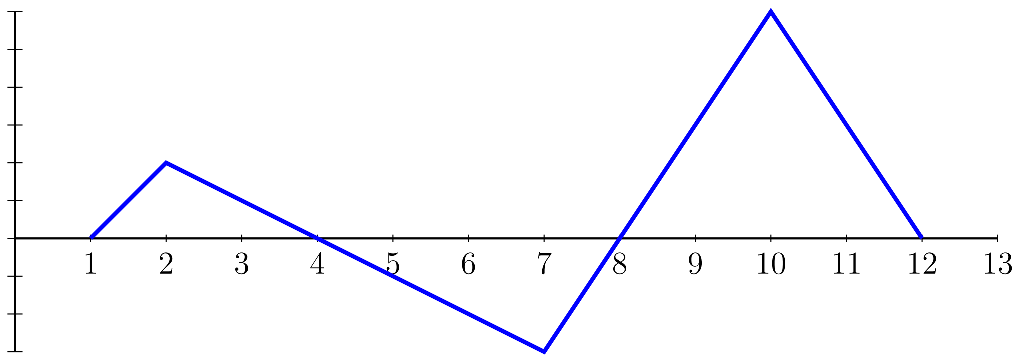
a)[4] Write a formula for velocity as a function of time t (for $0 \le t \le 15$).

b)[4] Write a definite integral which computes the distance traveled during the first six seconds.

c)[3] Will a left Riemann sum with $\Delta t = 1$ seconds for your answer to part (b) yield an under-estimate, exact value, or an over-estimate? Explain. (Note: this part does not ask you to do the actual left sum calculation.)

d)[4] Use the Fundamental Theorem of Calculus to compute exact value for your part (b) answer.

6.[10] Function f is graphed in the following figure.



Consider the following definite integrals — arrange them in order from smallest to largest.

Your answer can cite just the upper-case letters, e.g., respond R,T,Q,S if $R < T < Q < S$.

$$A = \int_1^4 f(x)\,dx \qquad B = \int_1^8 f(x)\,dx \qquad C = \int_1^{12} f(x)\,dx \qquad D = \int_5^5 f(x)\,dx$$

$$E = \int_4^8 f(x)\,dx \qquad F = \int_4^{12} f(x)\,dx \qquad G = \int_8^{12} f(x)\,dx$$

7. [15] Classify each of the following statements as **TRUE** or **FALSE**, then discuss each classification. If a statement is false, then either explain why it is false or revise it to be correct; if a statement is true, then provide supporting evidence. Note: 60% of the credit on each part of this problem is for an adequate discussion (**but** an irrelevant statement such as "2 + 1 = 3" does not qualify as a correct revision for any calculus statement).

a) If $f'(c) = 0$, then $f(x)$ has either a local minimum or a local maximum at $x = c$.

b) Every cubic polynomial has an inflection point.

c) Suppose f and g are continuous on $[2, 5]$. If f is increasing but g is decreasing on $[2, 5]$, then

$$\int_2^5 f(x)\,dx \neq \int_2^5 g(x)\,dx$$