Test 2 will be 28–Mar–2017 (the first Tuesday after Spring Break); its tasks will emphasize ideas in sections 2.3 through 4.2. Most of these problems were selected from previous versions of Test 2.

1. [10] Let $f(t)$ be the depth (in centimeters) of water in a tank at time t (in minutes).

   a) [2] What does the sign of $f'(t)$ tell us?

   b) [4] Explain the meaning of $\boldsymbol{f'(30) = 20}$ [include units]

   c) [4] Use the information in part (b), at time $t = 30$ minutes, to find the rate-of-change for depth in meters with respect to time in hours at time in hours.

2. [10] Let $g(x) = \ln(-3\,x)$.

   a) [2] What is the (maximal) domain of g?

   b) [2] What is the range of g?

   c) [6] Write an expression for $g'(x)$ [simplify your answer].

3. [15] Let $f(x) = 3 + 2\,e^{-0.5\,x}$

   a) [7] Find the *local linearization* of f at $x = 0$.
   (This is also called the *Tangent Line Approximation* for f near $x = 0$.)

   b) [5] Use your answer for part (a) to compute an approximation of $f(0.5)$.

   c) [3] Identify a feature of the graph of f (a sketch might be useful) that lets you decide whether your answer to part (b) is an over-estimate or an under-estimate for the true value of $f(0.5)$.

4. [12] The Bay of Fundy (Atlantic coast of Canada) is known for extreme tides. Depth (in meters) of water in the Bay of Fundy is modeled as a function of time t (hours after midnight) by the function

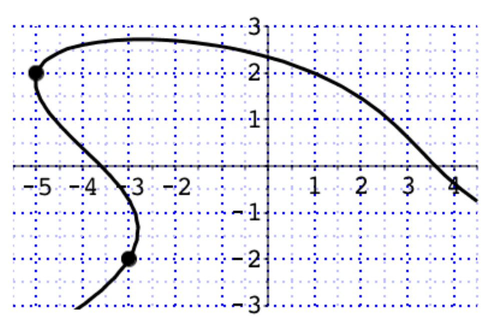
$$f(t) \;=\; 10 \;+\; 7.5\,\cos\left(\frac{\pi}{6}\,t\right)$$

   a) [4] Find derivative function(s) suitable to answer the items in part (b).

   b) [8] How quickly is the tide rising or falling (in meters-per-hour) at the following times?
   - 6:00 am
   - 9:00 am
   - noon
   - 6:00 pm

5. [18] The figure shows points (x, y) satisfying equation

$$x^2 \;+\; 2\,x\,y \;+\; y^3 \;=\; 13$$



   a) [8] Write an expression for $\dfrac{dy}{dx}$.
   Hint: use implicit differentiation.

   b) [4] Compute slope of the tangent at point $(-3, -2)$.

   c) [6] Write an equation for the tangent at point $(-5, 2)$.
   [Note: not the point in part (b).]

6. [10] This problem asks you to use differentiation facts to find some *antiderivatives*.

   a) Find a function f such that $f'(x) = e^{0.5\,x} + x$
   b) Find a function g such that $g''(x) = \sin(3\,x)$

7. [15] Classify each of the following statements as **TRUE** or **FALSE**, then discuss each classification. If a statement is false, then either explain why it is false or revise it to be correct; if a statement is true, then provide supporting evidence. Note: 60% of the credit on each part of this problem is for an adequate discussion (**but** an irrelevant statement such as "2 + 1 = 3" does not qualify as a correct revision for any calculus statement).

   a) If $g''(x) > 0$ for all x, then g' is a decreasing function.
   b) If f and g are two functions whose first and second derivatives exist, then
   $$\big(f \cdot g\big)'' \;=\; (f'' \cdot g) \;+\; (f \cdot g'')$$
   c) If $g(x) = f(-2\,x)$ and $f'(x) > 0$ for all x, then g is a decreasing function.

8. [10] Definition: **Relative-Rate-of-Change** for differentiable function g is $\mathbf{R}(g) \;=\; \dfrac{g'}{g}$

   a) Compute Relative-Rate-of-Change for $g(x) = e^{-3\,x}$. Simplify your answer.
   b) Show Relative-Rate-of-Change has the following property:
   $$\mathbf{R}(\,f \cdot g\,) \;=\; \mathbf{R}(f) \;+\; \mathbf{R}(g)$$
   Hint: Use the Product Rule, simplify stuff, then interpret the simplified result.

9. [15] Suppose g is differentiable such that $1 \le g'(x) \le 2$ for all x; also suppose $g(0) = 4$.

   a) [3] Explain why the information given above implies $g(3)$ can not be equal to 4.
   b) [4] Explain why the information given above implies $g(3)$ can not be equal to 20.
   c) [8] Show work which finds **Best Possible Bounds** for the value of $g(3)$.

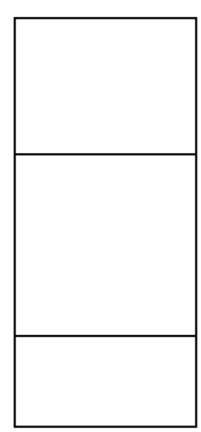
10. [10] Show work which locates all critical points and inflection points of $f(x) = x^4 - 18\,x^2 + 17$.

11. [15] Locate all extremes, local and global, of $p(x) = x \cdot \sqrt{|6 - x|}$ on closed interval $[-3, 10]$.

    Note: this function can also be written in the form $p(x) = \begin{cases} x \cdot \sqrt{6 - x} & \text{if} \quad x \le 6 \\ x \cdot \sqrt{x - 6} & \text{if} \quad x > 6 \end{cases}$

12. [10] Find area of the largest rectangle with one side on the x-axis and two upper corners on the graph of $y = 27 - x^2$.

13. [20] Three adjacent rectangular garden plots will share some fencing. This figure shows the desired configuration (with two internal fences, each shared by a pair of gardens).



   a) Suppose 100 feet of fencing is available for use on this project. What is the maximal area (total area of all three gardens) which can be enclosed?

   b) Suppose the three gardeners agree that they will be happy if total area of their gardens is 400 square-feet. What is the minimal length of fencing that will be needed?