We suggest you review all major topics we have studied since Test 3: sections 7.7-7.8, Chapter 8, sections 9.1–9.3, sections 10.1–10.2. Do not expect items on Test 4 to be copies of these practice problems. The test will include the following set of formulas.

Formulas

LAW OF COSINES: $c^2 = a^2 + b^2 - 2\,a\,b\cos C$ LAW OF SINES: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

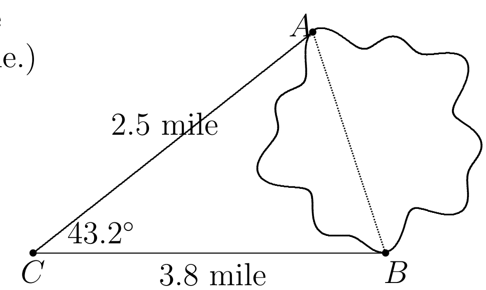
$\sin(x \pm t) = \sin x\cos t \pm \cos x\sin t$ $\cos(x \pm t) = \cos x\cos t \mp \sin x\sin t$

1. State the domain and range of $\arcsin = \sin^{-1}$, $\arccos = \cos^{-1}$, and $\arctan = \tan^{-1}$.

2. The ground crew for a hot air balloon is positioned 200 meters from the point of lift-off and monitors the ascent of the balloon.
   a) Find the height of the balloon when the ground crew's angle of observation is 70°.
   b) Express the height of the balloon h as a function of the crew's angle of observation θ.
   c) Find the angle of observation when the balloon is 100 meters high.
   d) Find a formula for the ground crew's angle of observation θ as a function of the height h [in meters] of the balloon.

3. The path of a satellite orbiting the Earth takes it directly over tracking stations A and B which are 50 miles apart. At 3pm, the satellite is West of both stations with the angle of elevation at A is 87.0° and the angle of elevation at B is 84.2°. At that time:
   a) How far is the satellite from station A?
   b) How high is the satellite above ground?

4. To find the distance across a small lake, a surveyor measured the angle and distances shown here. (This figure is not drawn to-scale.)
   a) Find the distance across the lake, i.e., between points A and B. (Have your calculator in degree mode.)
   b) Find angle A using the Law of Sines.
   c) Find angle A using the Law of Cosines.
   d) Which method (Sine Law or Cosine Law) for finding angle A do you prefer? Why?



5. a) Give Cartesian coordinates for $(r,\theta) = (2.3, 3.2)$: $(x,y) \approx (\_\_\_\_\_\_, \_\_\_\_\_\_)$
   b) Give polar coordinates for $(x,y) = (3,-4)$: $(r,\theta) \approx (\_\_\_\_\_\_, \_\_\_\_\_\_)$.
   c) Give polar coordinates (exact values) for $(x,y) = (-1,1)$: $(r,\theta) = (\_\_\_\_\_\_, \_\_\_\_\_\_)$.
   d) Write polar coordinate inequalities

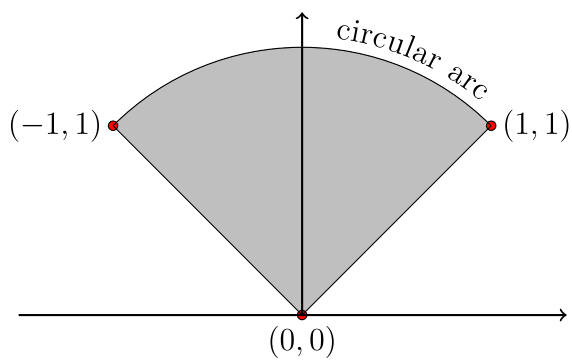
   $\_\_\_\_\_\_ \leq r \leq \_\_\_\_\_\_$

   and

   $\_\_\_\_\_\_ \leq \theta \leq \_\_\_\_\_\_$

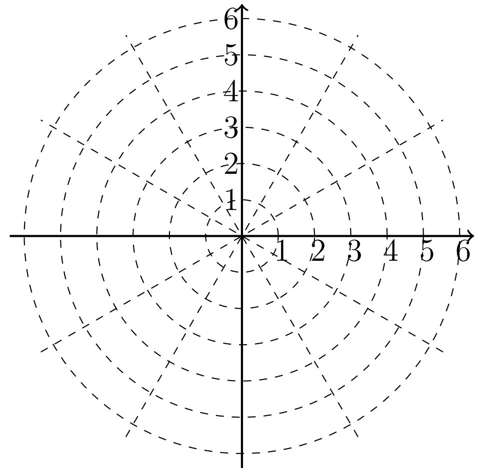
   to describe the shaded region shown at the right.

   Note: Cartesian coordinates for three points are shown in the figure.



6. Consider the following steps to graph the polar equation

$$r \;=\; 2 \;+\; 4\,\cos(\theta), \qquad 0 \le \theta \le 2\,\pi$$



   a) Sketch $y = 2 + 4\,\cos(x)$ for $0 \le x \le 2\,\pi$
   [create this (x, y) graph using Cartesian coordinates].
   b) Tabulate information about some points that are important on the polar curve. For example,
   - θ values where the polar curve goes through the origin
   - points where the polar curve is furthest from the origin
   - points where the polar curve crosses the x- or y-axis

   c) Sketch the polar curve using the polar coordinate grid.
   d) Mark each tabulated point with a big dot on the polar curve.

7. Solve these equations for θ in $[0, 2\pi]$. Give exact values in radians (in terms of π) when possible.
   a) $\sqrt{3}\,\tan(\theta) = -1$
   b) $\cos\theta \;+\; 3 \;=\; 1 \;-\; 5\,\cos\theta$

8. Graph the curve $y = -4\,\sin\left(\dfrac{\pi}{5}\,x\right) - 3.$ Use algebra to find exact value of one x-intercept. Then use the graph to approximate the *four smallest positive* x-intercepts.

9. An animal population oscillates during a 4-year period, starting at a low of 1000 animals, reaching a high of 3000 animals at end of the second year, and dropping back to 1000 at end of the fourth year.
   a) Sketch a graph to show how this population varies over a 10 year period.
   b) Find a formula for a sinusoidal function modeling this population t years after a lowest value.
   c) Find when the population will be around 1200 during the first 10 years; show in your graph.

10. Find all solutions (exactly) to the equation $2\,\cos^2 t \;=\; -3\,\sin t \,+\, 3$ for $0 \le t \le 2\pi$.

11. Consider an angle θ in the fourth quadrant such that $\cos\theta = \dfrac{5}{13}$. Use identities to find exact values for each of the following six expressions.

$\sin(\theta) =$	$\tan(\theta) =$
$\sec(\theta) =$	$\sin(2\pi - \theta) =$
$\cos(2\,\theta) =$	$\sin(2\,\theta - \pi) =$

12. Show how to use some identities together with Sine and Cosine values for 30°, 45°, 60° to compute exact values for $\sin(15°)$ and $\cos(165°)$.

13. Use sum and difference identities to prove the following identity: $\cos(2\pi - \theta) = \cos(\theta)$

14. Let $p(a)$ represent the weight, in pounds, of grass seed required to plant a lawn with area a. Let $c(s)$ represent the dollar cost to buy s pounds of grass seed.
   a) Which composition makes sense: $c\big(p(a)\big)$ or $p\big(c(s)\big)$?
   b) What does it represent?

15. Use algebra and theory about trigonometric functions to prove the following identities.
   a) $\dfrac{1 - \sin x}{\cos x} - \dfrac{\cos x}{1 + \sin x} \;=\; 0$
   b) $(\sin\theta + \cos\theta)^2 \;=\; 1 \;+\; \sin(2\,\theta)$