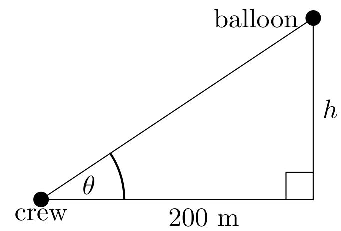
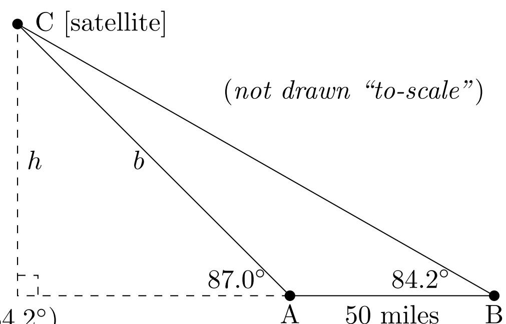
1.
   - $\arccos = \cos^{-1}$ has domain $[-1, 1]$ and range $[0, \pi]$
   - $\arcsin = \sin^{-1}$ has domain $[-1, 1]$ and range $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
   - $\arctan = \tan^{-1}$ has domain $(-\infty, \infty)$ and range $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$



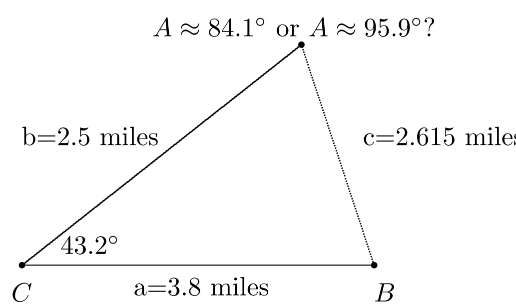
2. The ground crew for a hot air balloon is positioned 200 meters from the point of lift-off and monitors the ascent of the balloon. If h is the balloon's height and θ is the crew's angle of observation; then θ is a right-triangle angle with $\tan(\theta) = \dfrac{\text{Opp}(\theta)}{\text{Adj}(\theta)} = \dfrac{h}{200}$.
   a) When $\theta = 70°$, the balloon's height is $h = 200 \cdot \tan(70°) \approx 549.495$ meters.
   b) $\tan(\theta) = h/200$ implies $h = 200 \cdot \tan(\theta)$.
   c) When balloon's height is 100 meters, the crew's angle is $\theta = \tan^{-1}\left(\dfrac{100}{200}\right) = \tan^{-1}\left(\dfrac{1}{2}\right) \approx 26.565°$
   d) $\tan(\theta) = \dfrac{h}{200}$ and θ is an angle in a right triangle implies $\theta = \tan^{-1}\left(\dfrac{h}{200}\right)$.



3. The path of a satellite orbiting the Earth takes it directly over tracking stations A and B which are 50 miles apart. At 3pm, the satellite is West of both stations with the angle of elevation at A is 87.0° and the angle of elevation at B is 84.2°.

   Angle A in triangle ABC is $180° - 87.0° = 93°$; therefore angle C in that triangle is $180° - (93° + 84.2°) = 2.8°$.
   a) Law of Sines implies the satellite's distance from station A is $b = \dfrac{\sin(84.2°)}{\sin(2.8°)} \cdot 50 \approx 1018.31$ miles.
   b) Satellite's height above ground is $h = b \cdot \sin(87.0°) = \dfrac{\sin(84.2°)}{\sin(2.8°)} \cdot 50 \cdot \sin(87.0°) \approx 1016.91$ miles.

4. a) Apply Law of Cosines with side c being the distance across the lake between A and B. Therefore $c^2 = a^2 + b^2 - 2\,a\,b\,\cos(C) = 3.8^2 + 2.5^2 - 2 \cdot (3.8) \cdot (2.5) \cdot \cos(43.2°)$ implies $c \approx 2.6$ miles.



   b) Since A is the angle across the largest side, it could be an obtuse angle. $\sin(A) = \dfrac{3.8}{2.6} \cdot \sin(43.2°)$
   Using arcsine, $A \approx 84.1°$. Or is $A \approx 180° - 84.1°$?
   If you really want to use law of sines, use it to compute B instead. Since there is only one possible obtuse angle in a triangle, law of sines will work well for the acute angle B.
   $\sin(B) = \dfrac{2.5}{2.6} \cdot \sin(43.2°)$, so $B \approx 40.9°$.
   $A = 180° - 43.2° - B \approx 95.9°$.
   c) Law of Cosines: $a^2 = b^2 + c^2 - 2\,b\,c\,\cos(A)$ implies
   $$\cos(A) = \frac{b^2 + c^2 - a^2}{2\,b\,c} \approx \frac{2.5^2 + 6.839\,596\,079 - 3.8^2}{2 \cdot (2.5) \cdot (2.6)} \approx -0.103\,271$$
   which yields $A \approx \cos^{-1}(-0.103\,271) \approx 95.927°$.
   d) Work for part (c) is much simpler than any use of Law of Sines in part (b).
   If all three sides of a triangle are known, then Law of Cosines provides an unambiguous procedure to find any angle of the triangle. A key reason for that is the fact that the range of $\arccos = \cos^{-1}$ is the closed interval $[0°, 180°]$.

5. a) $(r, \theta) = (2.3, 3.2)$ implies $x = r\cos(\theta) = 2.3 \cdot \cos(3.2) \approx -2.296\,078$ and $y = r\sin(\theta) = 2.3 \cdot \sin(3.2) \approx -0.134\,261$. Therefore $(x, y) \approx (-2.296, -0.134)$.

b) Because $(x, y) = (3, -4)$ is a point in quadrant IV and the arctangent function evaluated at a negative number yields an angle in that quadrant, we can use polar coordinates

$$(r, \theta) = \left(\sqrt{3^2 + (-4)^2}, \tan^{-1}\left(\frac{-4}{3}\right)\right) \approx (5, -0.927\,295) \approx (5, -0.927).$$

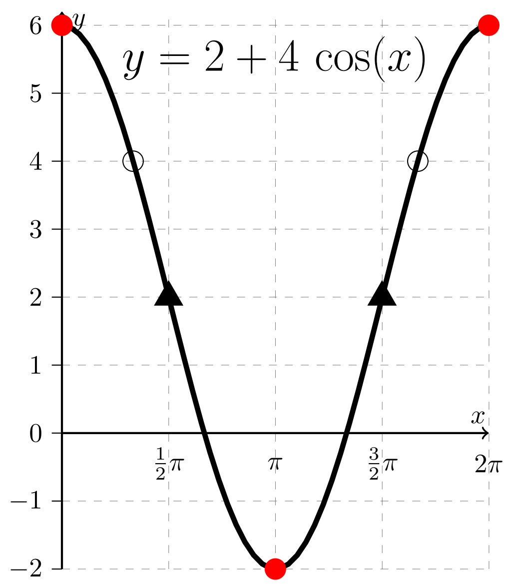
c) $(x, y) = (-1, 1)$ is in quadrant II. The two simplest polar coordinates for this point are

$$(r_1, \theta_1) = \left(\sqrt{(-1)^2 + 1^2}, \tan^{-1}\left(\frac{1}{-1}\right) + \pi\right) = \left(\sqrt{2}, \frac{3\,\pi}{4}\right)$$

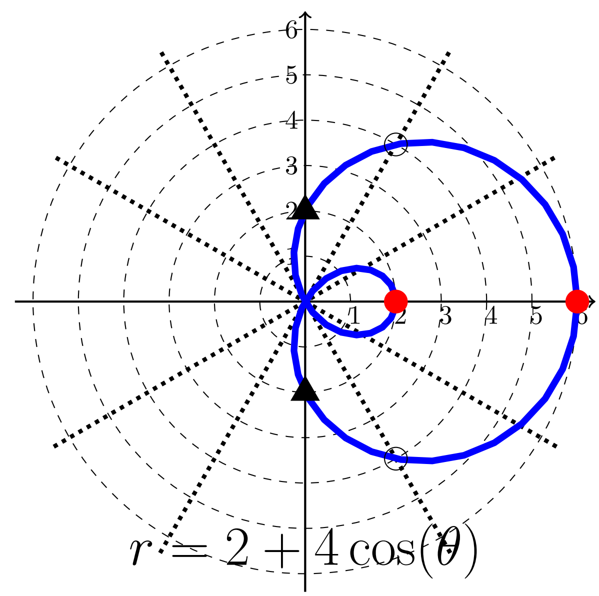
$$(r_2, \theta_2) = \left(-\sqrt{(-1)^2 + 1^2}, \tan^{-1}\left(\frac{1}{-1}\right)\right) = \left(-\sqrt{2}, \frac{-\pi}{4}\right)$$

d) One of the marked points, $(x, y) = (-1, 1)$, was analyzed in part (c). The polar inequalities for this part are $0 \le r \le \sqrt{2}$ and $\frac{\pi}{4} \le \theta \le \frac{3\,\pi}{4}$.

6. Cartesian graph is shown on the left, a table of interesting function values is in the middle, polar coordinate graph is on the right. The "interesting points" are marked with open circles, filled triangles, and red dots. The "inside loop" of the polar curve is traversed for $\frac{2\pi}{3} < \theta < \frac{4\pi}{3}$ [see where Cartesian graph is below x-axis].



t	$2 + 4\cos(t)$
0	6
$\pi/3$	4
$\pi/2$	2
$2\pi/3$	0
π	-2
$4\pi/3$	0
$3\pi/2$	2
$5\pi/3$	4
2π	6

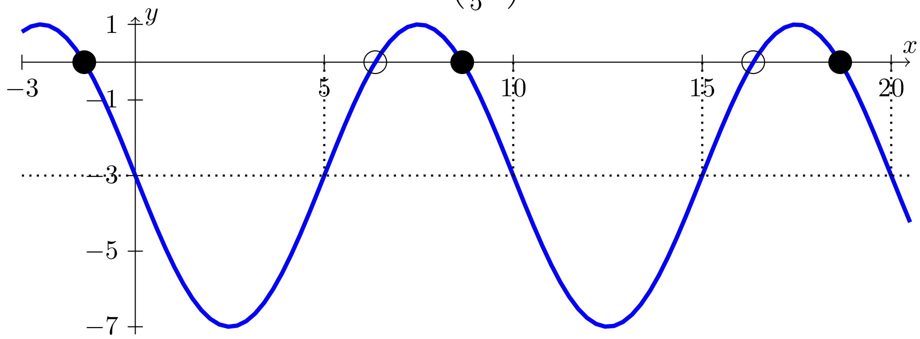


7. a) $\sqrt{3}\tan(\theta) = -1$ is equivalent to $\tan(\theta) = \frac{-1}{\sqrt{3}}$, but $\theta_0 = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{-\pi}{6}$ is not in $[0, 2\pi]$. The two solutions in that interval are $\theta_1 = \theta_0 + \pi = \frac{5\,\pi}{6}$ and $\theta_2 = \theta_0 + 2\,\pi = \frac{11\,\pi}{6}$.

b) $\cos\theta + 3 = 1 - 5\cos\theta$ is equivalent to $6\cos\theta = \cos\theta + 5\cos\theta = 1 - 3 = -2$ and $\cos\theta = \frac{-2}{6} = \frac{-1}{3}$. The smallest positive solution of $\cos\theta = \frac{-1}{3}$ is $\theta_1 = \cos^{-1}\left(\frac{-1}{3}\right) \approx 1.910\,633\,236$.

The reference angle for θ_1 is $\alpha = \pi - \theta_1$ and the next smallest positive solution of the equation is $\theta_2 = \pi + \alpha = 2\,\pi - \theta_1 \approx 4.372\,552\,071$.

8. This figure shows more than two periods of $f(x) = -4\sin\left(\frac{\pi}{5}x\right) - 3$.



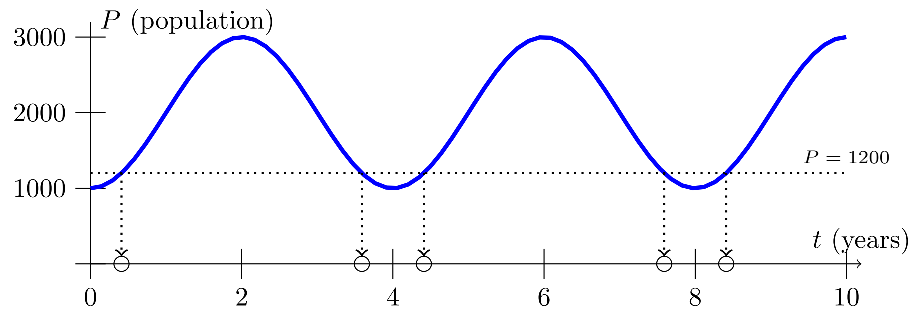
$f(x) = 0$ is equivalent to $\sin\left(\frac{\pi}{5}x\right) = \frac{-3}{4}$ which has an exact solution computed by

$$x_0 = \frac{5}{\pi}\cdot\sin^{-1}\left(\frac{-3}{4}\right) \approx -1.349\,732\,719$$

Although x_0 is not positive, period of f is $\frac{2\pi}{\pi/5} = 2\cdot 5 = 10$ implies $x_2 = x_0 + 10 \approx 8.650$ and $x_4 = x_0 + 20 \approx 18.650$ are zeros of f. The graph has solid dots at x_0 and those two small positive x-intercepts. Because this graph is symmetric about its midline, $x_1 = 5 + (10 - x_2) = 5 - x_0 \approx 5 + 1.349\,732\,719 \approx 6.350$ and $x_3 = x_1 + 10 = 15 - x_0 \approx 16.350$ are also zeros of f; these small positive x-intercepts are marked on the x-axis with an open circle.

9. *An animal population oscillates during a 4-year period, starting at a low of* 1000 *animals, reaching a high of* 3000 *animals at end of the second year, and dropping back to* 1000 *at end of the fourth year.*

   a) This graph shows how this animal population varies over a 10 year period.



   b) The graph is for function $P = f(t) = 2000 - 1000\cos\left(\frac{\pi}{2}t\right)$.

   c) Smallest positive solution of $f(t) = 1200$ is $t_1 = \frac{\cos^{-1}\big((2000-1200)/1000\big)}{\pi/2} = \frac{2\cos^{-1}(4/5)}{\pi} \approx 0.41$ years. The other solutions in $[0, 10]$ are $t_2 = 4 - t_1 \approx 3.59$, $t_3 = t_1 + 4 \approx 4.41$, $t_4 = t_2 + 4 \approx 7.59$, and $t_5 = t_1 + 8 = t_3 + 4 \approx 8.41$.

10. Identity $\sin^2 + \cos^2 = 1$ implies equation $2\cos^2 t = -3\sin t + 3$ is equivalent to

$$\begin{aligned} 0 &= -2\cos^2 t - 3\sin t + 3 \\ &= -2\cdot\left(1 - \sin^2 t\right) - 3\sin t + 3 = -2 + 2\sin^2 t - 3\sin t + 3 \\ &= 2\sin^2 t - 3\sin t + 1 \\ &= (2\sin t - 1)\cdot(\sin t - 1) \end{aligned}$$

$$\sin t = \frac{1}{2} \qquad \text{OR} \qquad \sin t = 1$$

The only solutions of $\sin t = 1/2$ in interval $[0, 2\pi]$ are $\pi/6$ and $5\pi/6$ and the unique solution in $[0, 2\pi]$ of $\sin t = 1$ is $\pi/2$.

11. If θ ends in the fourth quadrant, then $\sin\theta$ is negative. If $\cos\theta = \dfrac{5}{13}$, then

$$\begin{aligned}
\sin(\theta) &= -\sqrt{1-\cos^2\theta} = -\sqrt{1-\left(\frac{5}{13}\right)^2} = \frac{-\sqrt{13^2-5^2}}{13} = \frac{-\sqrt{144}}{13} = \frac{-12}{13} \\
\tan(\theta) &= \frac{\sin\theta}{\cos\theta} = \frac{-12/13}{5/13} = \frac{-12}{5} \\
\sec(\theta) &= \frac{1}{\cos\theta} = \frac{1}{5/13} = \frac{13}{5} \\
\sin(2\pi-\theta) &= \sin(-\theta) = -\sin(\theta) = \frac{12}{13} \\
\cos(2\,\theta) &= 2\cos^2\theta - 1 = 2\cdot\left(\frac{5}{13}\right)^2 - 1 = \frac{50}{169} - 1 = \frac{-119}{169} \\
\sin(2\,\theta-\pi) &= \sin(2\,\theta)\cdot\cos(\pi) - \cos(2\,\theta)\cdot\sin(\pi) = \sin(2\,\theta)\cdot(-1) - 0 \\
&= -\sin(2\,\theta) \\
&= -2\cdot\sin(\theta)\cdot\cos(\theta) \\
&= -2\cdot\left(\frac{-12}{13}\right)\cdot\left(\frac{5}{13}\right) = \frac{120}{169}
\end{aligned}$$

12. Note that $15 = 45-30 = 60-45$ and $165 = 120+45 = 135+30$.

$$\begin{aligned}
\sin(15^\circ) &= \sin(45^\circ - 30^\circ) \\
&= \sin(45^\circ)\cdot\cos(30^\circ) - \cos(45^\circ)\cdot\sin(30^\circ) \\
&= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4} \\
\cos(165^\circ) &= \cos(120^\circ + 45^\circ) \\
&= \cos(120^\circ)\cdot\cos(45^\circ) - \sin(120^\circ)\cdot\sin(45^\circ) \\
&= \frac{-1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{-\sqrt{2}-\sqrt{6}}{4}
\end{aligned}$$

13. $\cos(2\pi-\theta) = \cos(2\pi)\cos(\theta) - \sin(2\pi)\sin(\theta) = (1)\cos(\theta) - (0)\sin(\theta) = \cos\theta$

14. *Let $p(a)$ represent the weight, in pounds, of grass seed required to plant a lawn with area a. Let $c(s)$ represent the dollar cost to buy s pounds of grass seed.*

   a) $a \longrightarrow w = p(a) \longrightarrow c(w) = c\big(p(a)\big)$: Composition of these functions in the other order is nonsense. Output from $c(s)$ is dollars but function p needs its input to be area of some lawn.

   b) A lawn with area a requires $w = p(a)$ pounds of grass seed and that weight of seed will cost $c(w) = c\big(p(a)\big)$ dollars to buy. $\text{Cost}(a) = c\big(p(a)\big)$ is the cost to seed a lawn whose area is a.

15. a) Start with algebra of fractions, use the algebraic identity $(a-b)\cdot(a+b) = a^2-b^2$, finish by using the Pythagorean Identity ($\sin^2 + \cos^2 = 1$) to get a numerator that is $1-1=0$.

$$\begin{aligned}
\frac{1-\sin x}{\cos x} - \frac{\cos x}{1+\sin x} &= \frac{(1-\sin x)\cdot(1+\sin x) - (\cos x)\cdot(\cos x)}{(\cos x)\cdot(1+\sin x)} \\
&= \frac{(1^2-\sin^2 x) - \cos^2 x}{\text{stuff}} \\
&= \frac{1-(\sin^2 x + \cos^2 x)}{\text{stuff}} = \frac{1-1}{\text{stuff}} = \frac{0}{\text{stuff}} = 0
\end{aligned}$$

   b) Begin with $(a+b)^2 = a^2 + 2ab + b^2$, use the Pythagorean identity, finish with the Sine double-angle formula.

$$\begin{aligned}
(\sin\theta + \cos\theta)^2 &= \sin^2\theta + 2\cdot(\sin\theta)\cdot(\cos\theta) + \cos^2\theta \\
&= \big(\sin^2\theta + \cos^2\theta\big) + 2\cdot(\sin\theta)\cdot(\cos\theta) \\
&= 1 + 2\cdot(\sin\theta)\cdot(\cos\theta) \\
&= 1 + \sin(2\,\theta)
\end{aligned}$$