4.5:38) (a) The two-year balances for these investments are computed below

$$A = 875 \cdot \left(1 + \frac{0.135}{365}\right)^{(365)(2)} \approx 875 \cdot (1.309\,899\,056) \approx \$1146.16$$

$$B = 1000 \cdot e^{(0.067)(2)} \approx 1000 \cdot (1.143\,392\,820) \approx \$1143.39$$

$$C = 1050 \cdot \left(1 + \frac{0.045}{12}\right)^{(12)(2)} \approx 1050 \cdot (1.093\,990\,118) \approx \$1148.69$$

b) The two-year factors which are computed above show A is best (two-year factor ≈ 1.310) while C is worst (two-year factor ≈ 1.094). The same conclusion is reached by comparing the effective annual rates:

$$A \;:\; \left(\left(1 + \frac{0.135}{365}\right)^{365} - 1\right) \cdot 100\% \approx 14.45\% \text{ per year}$$

$$B \;:\; \left(e^{0.067} - 1\right) \cdot 100\% \approx 6.93\% \text{ per year}$$

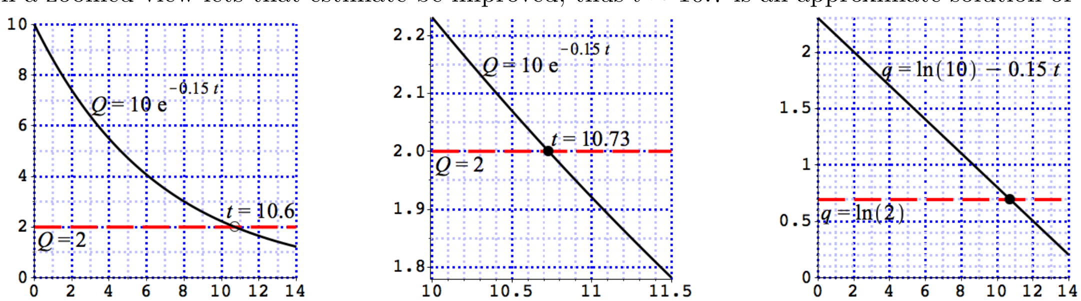
$$C \;:\; \left(\left(1 + \frac{0.045}{12}\right)^{12} - 1\right) \cdot 100\% \approx 4.59\% \text{ per year}$$

Note: Since starting balances were different, it would be a mistake to identify best & worst by comparing final balances.

5.1:54) The lefthand figure (see below) shows a graph of $Q = f(t) = 10\,e^{-0.15\,t}$.

a) The initial value of Q is $f(0) = 10$. The continuous decay rate is $0.15 = 15\%$ (per time unit).

b) Horizontal line $Q = 2$ meets the graph of $Q = f(t) = 10\,e^{-0.15\,t}$ at a point which is circled in the left and middle figures. Horizontal coordinate of that point in the left figure looks about midway between 10 and 11 but the middle figure with a zoomed view lets that estimate be improved; thus $t \approx 10.7$ is an approximate solution of $Q = f(t) = 2$.



c) Righthand figure shows intersection of straight lines $q = \ln(2)$ and $q = \ln\left(10\,e^{-0.15\,t}\right) = \ln(10) - 0.15\,t$.
A logarithm finds the exact solution of $Q = f(t) = 2$ The following equations are equivalent

$$10\,e^{-0.15\,t} = 2$$

$$e^{-0.15\,t} = \frac{2}{10} = 0.2$$

$$-0.15\,t = \ln\left(e^{-0.15\,t}\right) = \ln(0.2)$$

$$t = \frac{\ln(0.2)}{-0.15} \approx 10.729\,586\,083 \approx 10.730$$

5.2:56) The following table presents the doubling time predicted by the Rule of 70 together with (rounded) results of exact computations. If percent rate R is annual, then the annual growth factor is $b = 1 + (R/100)$, the t-year growth factor is b^t and the exact doubling time is $\dfrac{\ln(2)}{\ln(b)} = \dfrac{\ln(2)}{\ln(1+R/100)}$. On the other hand, if R is the continuous rate (in percent), then the t-year growth factor is $e^{k\,t}$ where $k = R/100$ and the exact doubling time is $\dfrac{\ln(2)}{k} = \dfrac{\ln(2)}{R/100}$.

rate R (%)	1	2	5	7	10
predicted doubling time (years)	$70/1 = 70$	$70/2 = 35$	$70/5 = 14$	$70/7 = 10$	$70/10 = 7$
exact doubling time (years) if rate is annual	69.661	35.003	14.207	10.245	7.273
exact doubling time (years) if rate is continuous	69.315	34.657	13.863	9.902	6.931

Caution: do not use the Rule of 70 for annual compounding of growth rates much larger than ten percent. For example, the Rule of 70 predicts 2 years if $R = 35\%$ but the two-year factor $(1+0.35)^2 = 1.8225$ is quite a bit less than doubling.

Note: in the continuous context, the doubling time is $\dfrac{\ln(2)}{R/100} = \dfrac{100 \times \ln(2)}{R} \approx \dfrac{69.315}{R} \approx \dfrac{70}{R}$.

5.3:36) a) The average rate of change for $\log(x)$ on interval $[10, 100]$ is

$$\frac{\log(100) - \log(10)}{100 - 10} = \frac{\log(10^2) - \log(10^1)}{100 - 10} = \frac{2-1}{90} = \frac{1}{90}$$

b) The average rate of change for 10^x on interval $[1, 2]$ is

$$\frac{10^2 - 10^1}{2-1} = \frac{100-10}{2-1} = \frac{90}{1} = 90$$

c) These two functions, base-ten logarithm and base-ten exponential, are inverse functions. Points $(10, 1)$ and $(100, 2)$ are on the graph of the logarithm while points $(1, 10)$ and $(2, 100)$ are on the graph of the exponential. Because those points are related by swapping the x- and y-values, the slopes between those points are reciprocals of each other.

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written homework for week 5: Notes About Grading and Partial Credit

4.5:38) 5 = 3 + 2 points

- 3 points for two-year balances: 1 point for each
- 2 points for using either *two-year growth factor* or *effective annual rate* to identify best and worst

5.1:54) 5 = 2 + 1 + 2 points (graph for part b is optional)

a) 2 points: 1 each for initial value and continuous decay rate (but only 0.5 each if mere number without interpretation in context)

b) 1 point for an estimate near 10.7 (within 0.3 is sufficient)

c) 2 points:

- 1 point for work showing use of the logarithm property $\log(b^t) = t \cdot \log(b)$,
- 1 point for either the exact answer or a good numerical approximation

5.2:56) 5 = 2 + 3 points

- **Rule of 70**: 2 points for at least three of the $70/R$ computations; 1 point for two or fewer
- **exact doubling time**: 3 points — preferably 1 for formula and 2 for at least two computations but could be 3 points for computing all five exact doubling times (using either context)
- Note: a student paper does not need to include an explicit statement about choice of annual or continuous context.
- Although size ($1000) of the initial investment is irrelevant to questions about "doubling time", allow full credit if analysis is stated in terms of dollar amounts.

5.3:36) 5 = 1+1+3 points

a) 1 point for computation

b) 1 point for computation

c) 1 for mentioning relation between points on the graphs,
1 for observing the two functions are inverses of each other,
1 for remarking the two slopes (average rates) are reciprocals of each other