M 151 – Precalculus Review – Spring 2017 – Final Exam

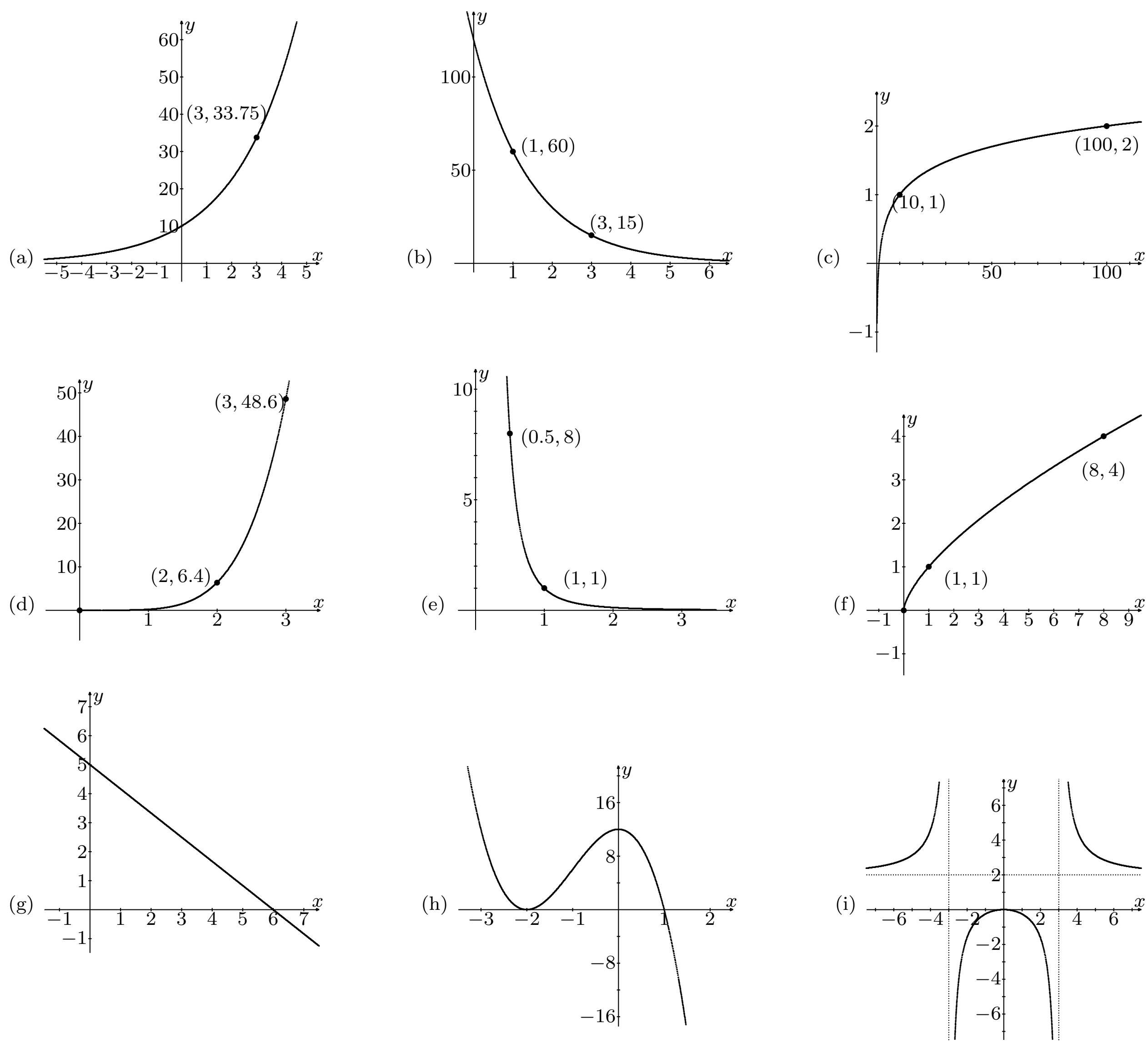
The final exam will be Tuesday, May 9 from 6 pm to 8 pm: in Urey Lecture Hall (ULH)

We suggest you use your previous tests, some of the current worksheets, the online problems and the following additional problems for identifying the topics you are having difficulties with. As you review, prepare a 4×6 card with notes and formulas to bring to the final exam. We will provide a list with the formulas which were given in test 4.

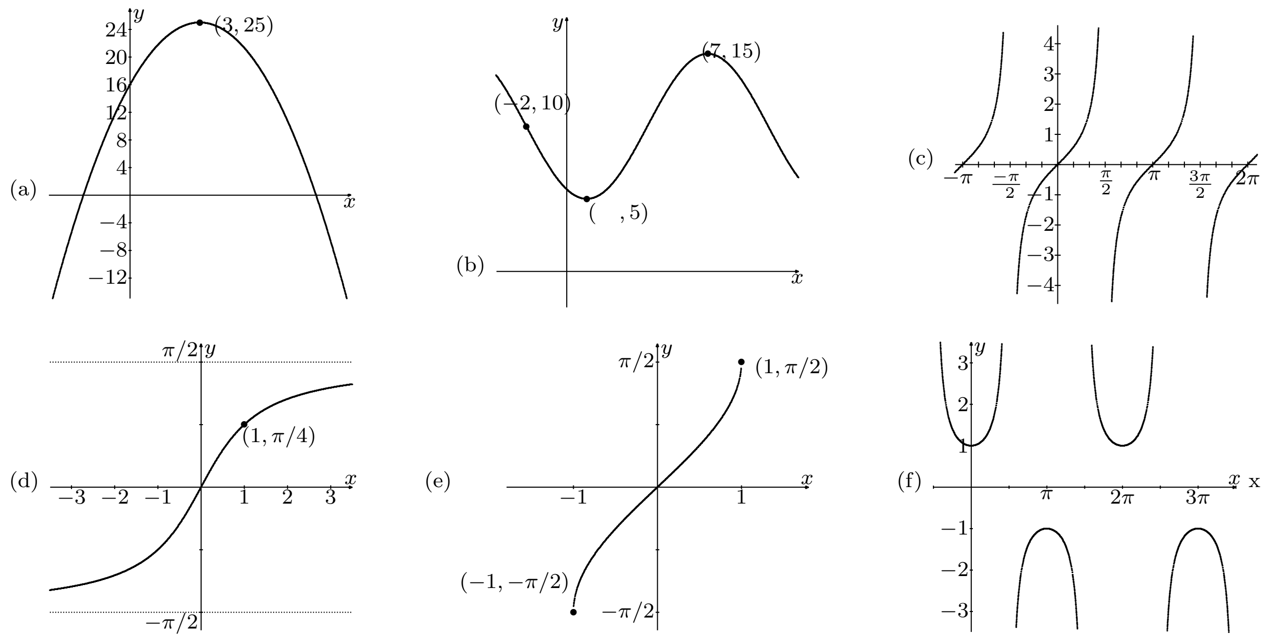
1. A flight costs \$10,000 to operate, regardless of the number of passengers. Each ticket costs \$127. Express the profit P, as a linear function of the number of passengers, n, on the flight.

2. Draw the graph of the following function. Find $f(f(-5))$. $f(x) = \begin{cases} -x, & \text{for } x < 3, \\ 2, & \text{for } x \geq 3 \end{cases}$

3. Classify the functions which generated these graphs as linear, exponential, logarithmic, power (negative values of x were excluded), polynomial or rational. Note: The x-axis is a horizontal asymptote for functions in (a), (b) and (e). The y-axis is a vertical asymptote for functions (c) and (e). The point $(0, 0)$ is on the graphs of functions (d), (f) and (i). Find possible equations for these functions.



4. Find possible formulas for these functions; (b) and (f) are periodic; (d) has 2 hor. asymptotes.



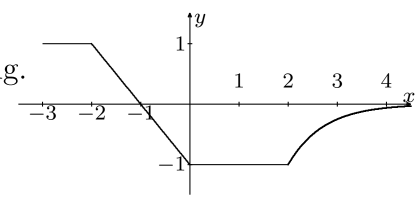
5. Complete the following table with the functions f, g and h given that (a) f is an even function

(b) g is an odd function

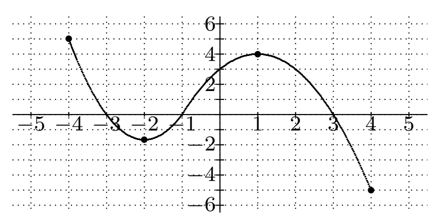
(c) h is the composition $h(x) = g(f(x))$.

x	$f(x)$	$g(x)$	$h(x)$
−4	−2	−2	
−2	4	4	
0	0	0	
2			
4			

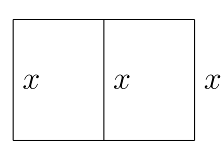
6. Use the graph of f, given below, to graph each of the following. Label any intercepts or asymptotes that can be determined.
(a) $y = f(x-4)+3$, (b) $y = 2f(-x)$, (c) $y = 5 - f(x)$



7. Assume that $y = f(x)$ has domain$[-4, 4]$; its graph is given.
   (a) Find $f(2)$ and $f(f(1))$.
   (b) Solve the equation $f(x) = 0$.
   (c) Does this function have an inverse?



8. A rancher needs to enclose <u>two adjacent rectangular corrals</u>, one for cattle and one for sheep. If 180 yd of fencing is available, what are the dimensions of the corral of largest total area which can be enclosed? (Show graph of the area function.)



9. An object is thrown vertically upward with an initial velocity of 40 ft/sec from the top of a 176-foot high building. The height of the object t seconds after it is thrown can be modeled by the equation $h = f(t) = 176 + 40t - 16t^2$. When does the object reach maximum height? What is its maximum height? How long does the object stay in the air?

10. Classify each of the following statements as **True** or **False** and discuss each classification.
    (a) The function in question above, $h = f(t) = 176 + 40t - 16t^2$, has an inverse $t = f^{-1}(h)$. For example, $f^{-1}(152) = 3$, that is, if we know that the object is 152 ft above the ground, this function informs us that this happened 3 seconds after the object is thrown.
    (b) The graph of $g(x) = f(x - 1)$ is obtained by shifting the graph of f one unit to the left.
    (c) If $n = f(A)$ is the number of angels that can dance on the head of a pin whose area is A millimeters, then $f(10) = 100$ tells us that 10 angels can dance on the head of a pin whose area is 100 square millimeters.
    (d) The world's population in 1999 was about 6 billion, and in 2009 it was about 6.8 billion. Assume this trend involves exponential growth and that it continues. We then estimate the world population to be about 7.7 billion in 2019.
    (e) If A and B are positive numbers, then $\log(A + B) = \log(A) \cdot \log(B)$.
    (f) On a circle of radius 10 cm , the length of an arc cut off by an angle $60°$ is given by $s = r\theta = 600$ cm.
    (g) If the half-life of a radioactive substance is 5 hours, then there will be a fourth of the substance left in 7.5 hours.
    (h) The graph of any power function passes through the origin.

11. Let $f(x) = x^2 - 3x$. Compute the average rate of change of f in the interval $[1, 5]$.

12. Consider the quadratic function defined by $f(x) = 4x^2+8x-21$. Complete the square to write $f(x)$ in the form $f(x) = a(x-h)^2+k$. Factor $f(x)$ to write it in the form $f(x) = a(x-b)(x-c)$. Find the vertex, the zeros and the y-intercept of this function. Find the interval where this function is decreasing. Find the x-interval(s) on which the values of this function are positive.

13. Consider the polynomial function $p(x) = -2x^5 + 8x^4 - 8x^3$. Find all the zeros (with multiplicities) and factor $p(x)$ completely. Sketch the graph of p ; show zeros, intercepts, and end-behavior. bf Estimate the interval(s) where $p(x)$ is increasing.

14. Let $f(x) = \dfrac{2x^2 + 5x + 3}{9x^2 - 4} = \dfrac{(2x + 3)(x + 1)}{(3x - 2)(3x + 2)}$. Write an equation for each vertical and horizontal asymptote (if any), find all x- and y-intercepts (if any). Describe the end-behavior of this function: if $x \to \infty$ then $y \to$ ______ and if $x \to -\infty$ then $y \to$ ______.

15. Which function dominates as $x \to \infty$?
    Graph a ratio of the two functions (for each pair) to look at the long run behavior as $x \to \infty$.
    (a) $y = 300 + 20x$ or $y = 10(2^x)$ (b) $y = 5x^{1.2}$ or $y = 1000x^{1.15}$ (c) $y = 3.6x^{12}$ or $y = 1.2^x$

16. The half-life of iodine-123 is about 13 hours. You begin with 50 grams of iodine-123. How much iodine-123 remains after about 26 hours? Write an equation that gives the amount of iodine-123 remaining after t hours. Determine the number of hours needed for your sample to decay to 10 grams.

17. Driving at 55mph, it takes approximately 3.5 hours to drive from Long Island to Albany, NY. Is the time the drive takes directly or inversely proportional to the speed? Write a formula for the proportion. To get to Albany in 3 hours, how fast would you have to drive?

18. Find a formula "by hand" for a linear, an exponential and a power function through the points $(4, 70)$ and $(9,\ 157.5)$.

19. The following are exponential functions. Rewrite them in the form $y = ab^t$. For each, identify a, b, and r, the growth (or decay) rate. A table with $x = 0$ and $x = 1$ may help.

    (a) $30e^{0.2x}$ (b) $75e^{-0.08x}$ (c) $36(1.2^{x-1})$ (d) $36(0.8^{-x+1})$ (e) $36(1.2^{2x})$ (f) $36(0.6^x)$

20. The following are power functions. Rewrite them in the form $y = kx^p$. For each, identify k and p. A graph with $x = 0$, $x = 1$ and $x = 2$ will help identify whether the function is concave up or concave down, increasing or decreasing (allowing you to check if you got the correct p).

    (a) $30\sqrt{4x}$ (b) $\dfrac{75}{3x^2}$ (c) $5\sqrt[3]{8x^2}$ (d) $\dfrac{75}{\sqrt[4]{81x^3}}$

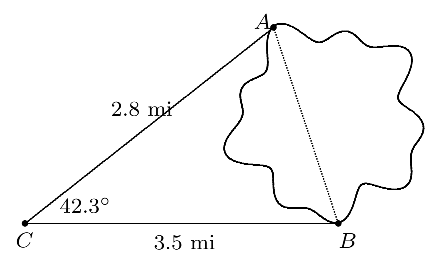
21. Classify each of the following functions as linear (L), quadratic (Q), exponential(E), power(Pwr), polynomial (Pol), rational (R) or neither (N). Choose all that apply.

    (a) $5x$ (b) $\dfrac{1}{3}x^2$ (c) $5x + \dfrac{1}{3}x^2$ (d) $5\sqrt{x} + \dfrac{1}{3}x^2$

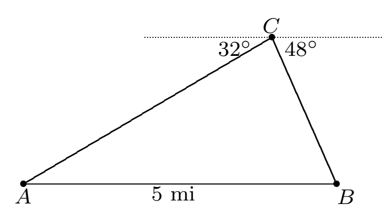
    (e) $\dfrac{3}{x^2}$ (f) $5 + \dfrac{3}{x^2}$ (g) $3.4x^{1.2}$ (h) $3.4(1.2)^x$

22. Estimate the length of the path connecting two points in the Equator which are 1° apart. (The radius of the Earth is approximately 3960 miles.)

23. To find the distance across a small lake, a surveyor has taken the measurements shown. Find the distance across the lake (the distance between points A and B) using this information.



24. A pilot (at point C) is flying over a straight highway. She determines the angles of depression to two mileposts A and B, 5 miles apart, to be 32° and 48°, as shown in the figure. Find the distance of the plane from point A, then find the elevation of the plane.



25. Let θ be in quadrant II with $\sin(\theta) = 0.6$.

    Use identities to find $\cos(\theta)$, $\tan(\theta)$, $\sin(2\theta)$, and $\cos(\theta + \pi/4)$.

26. The Brown County Ferris Wheel has a diameter of 50 meters and completes one full revolution every two minutes. When you are at the lowest point on the wheel, you are still 5 meters above the ground. Assuming you board the ride at $t = 0$ seconds, sketch a graph of your height, $h = f(t)$, as a function of time in minutes. What are the amplitude, midline and period of the function? Find a formula for this function. Find how long one spends above 30 meters within one turn.

27. Solve in $[0, 2\pi]$: $\cos x + 2\cos^2 x = 0$

28. Graph the polar curve $r = 8 - 12\cos(\theta)$ for $0 \leq \theta \leq 2\pi$.