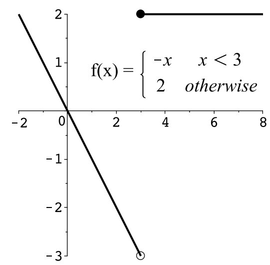
1) Profit for a flight with n passengers is $P(n) = 127 \cdot n \; - \; 10000$ dollars.

2) $f(-5) = -(-5) = 5$ and $f\Big(f(-5)\Big) = f(5) = 2$



3)

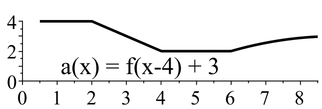
type	expression
exponential, increasing	$a(x) = 10\ (1.5^x)$
exponential, decreasing	$b(x) = 120\ (0.5^x)$
logarithmic	$c(x) = \log(x)$
power (positive integer), increasing	$d(x) = 0.2\,x^5$
power (negative integer), decreasing	$e(x) = x^{-3}$
power (fractional, less than one)	$f(x) = x^{2/3}$
linear, decreasing	$g(x) = (-5/6)\,x + 5$
polynomial (cubic)	$h(x) = -3\,(x+2)^2\,(x-1)$
rational function	$i(x) = \dfrac{2\,x^2}{(x+3)\,(x-3)}$

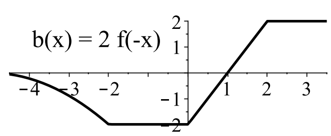
4)

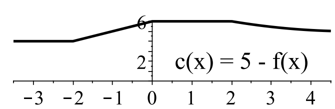
part	important information	type	expression
A	vertex at $(3, 25)$, y-intercept is 16	quadratic polynomial	$a(x) = 25 - (x-3)^2 = 16 + 6\,x - x^2$
B	period $= \dfrac{7-(-2)}{3/4}$, amp $= \dfrac{15-5}{2}$, mid $= 10$	sinusoidal	$B(x) = 10 \; + \; 5\,\cos\left(\dfrac{\pi}{6}\,(x-7)\right)$
C	vertical asymptotes at odd multiples of $\pi/2$	tangent function	$C(x) = \tan(x)$
D	inverse of 'middle part' of C	inverse tangent	$D(x) = \arctan(x) = \tan^{-1}(x)$
E	inverse of 'middle part' of sine	inverse sine	$E(x) = \arcsin(x) = \sin^{-1}(x)$
F	period is 2π, y-intercept is 1, unbounded	**periodic**	$F(x) = \dfrac{1}{\cos(x)} = \sec(x)$

5) $f(x)$ column: $-2, 4, 0, 4, -2$; $g(x)$ column: $-2, 4, 0, -4, -(-2) = 2$; $h(x)$ column: $g(-2) = 4, g(4) = 2, g(0) = 0, 2, 4$

6)







7) a) $f(2) = 3$ and $f\left(f(1)\right) = f(4) = -5$

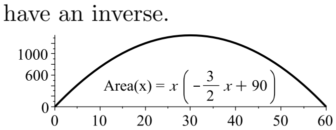
b) $f(x) = 0$ if and only if x is -3, -1, or 3

c) Answer to part (b) shows this function fails the "Horizontal Line Test" for $y = 0$, hence it does NOT have an inverse.

8) Let w be total width of the adjacent corrals. The fencing is $3\,x + 2\,w = 180$ feet.

Combined area is $A(x) = x \cdot w = x \cdot \dfrac{180 - 3\,x}{2} = 1350 - \dfrac{3}{2}\,(x-30)^2$ square-feet.

Maximal area of 1350 square-feet attained by choosing $x = 30$ feet and $y = 45$ feet.



9) $h(t) = 176 + 40\,t - 16\,t^2 = 201 - 16\left(t - \dfrac{5}{4}\right)^2$. Maximal height is $h\left(t = \dfrac{5}{4}\text{ sec}\right) = 201$ feet; object hits the ground at positive solution of $h(t) = 0$, i.e., $t = \dfrac{5}{4} + \sqrt{\dfrac{201}{16}} \approx 4.79$ seconds.

10) a) **FALSE**: $f(0) = 176 = f(2.5)$, the function is not one-to-one.

b) **FALSE**: Shift graph of f **right** by one to get graph of $g(x) = f(x-1)$.

c) **FALSE**: $f\left(10\text{ mm}^2\right) = 100$ angels can dance on a pin with area 10 square-millimeters.

d) **TRUE**: Ten-year growth factor is $6.8/6$ and $6 \cdot (6.8/6)^2 \approx 7.7067 \approx 7.7$

e) **FALSE**: $\log(1+2) = \log(3) > 0$ but $\log(1) \cdot \log(2) = 0 \cdot \log(2) = 0$

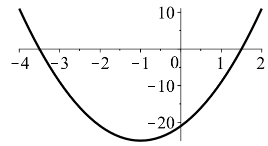
f) **FALSE**: Arc-length formula $s = r \cdot \theta$ presumes angle θ is measured in radians.

g) **FALSE**: If half-life is 5 hours, then one-quarter will remain after $5 + 5 = 10$ hours.

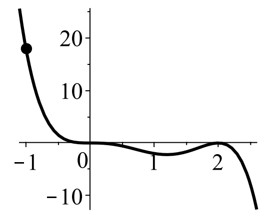
h) **FALSE**: If power p is positive, then $0^p = 0$; but $x^0 = 1$ is constant and x^{negative} is undefined at $x = 0$.

11) $f(1) = 1^2 - 3 \cdot 1 = -2$ and $f(5) = 5^2 - 3 \cdot 5 = 10$. Average rate of change for f on interval $[1, 5]$ is slope of the line through points $\big(1, f(1)\big) = (1, -2)$ and $\big(5, f(5)\big) = (5, 10)$.

$$\frac{f(5) - f(1)}{5 - 1} = \frac{10 - (-2)}{5 - 1} = \frac{12}{4} = 3$$



12) $4\,x^2 + 8\,x = 4 \cdot \left(x^2 + 2\,x\right)$ and $x^2 + 2\,x = (x+1)^2 - 1$, therefore
$f(x) = 4\,x^2 + 8\,x - 21 = 4\,(x+1)^2 - 25 = (2\,x + 7) \cdot (2\,x - 3)$.
y-intercept is -21, vertex of graph is $(-1, -25)$, x-intercepts are $-1 - \frac{5}{2} = \frac{-7}{2}$ and $-1 + \frac{5}{2} = \frac{3}{2}$.
This function is decreasing on interval $(-\infty, -1)$; its values are positive on $(-\infty, -7/2) \cup (3/2, \infty)$.



13) $p(x) = -2\,x^5 + 8\,x^4 - 8\,x^3 = -2\,x^3 \cdot \left(x^2 - 4\,x + 4\right) = -2\,x^3 \cdot (x-2)^2$.
The zeros of p are 0 (multiplicity 3) and 2 (multiplicity 2), y-intercept is 0, $p(-1) = 18$,
this function increases on the interval $[6/5, 2]$ and decreases on $(-\infty, 6/5] \cup [2, \infty)$,
it attains a local maximum at the point $(2, 0)$, $\lim_{x \to -\infty} p(x) = \infty$ and $\lim_{x \to \infty} p(x) = -\infty$.

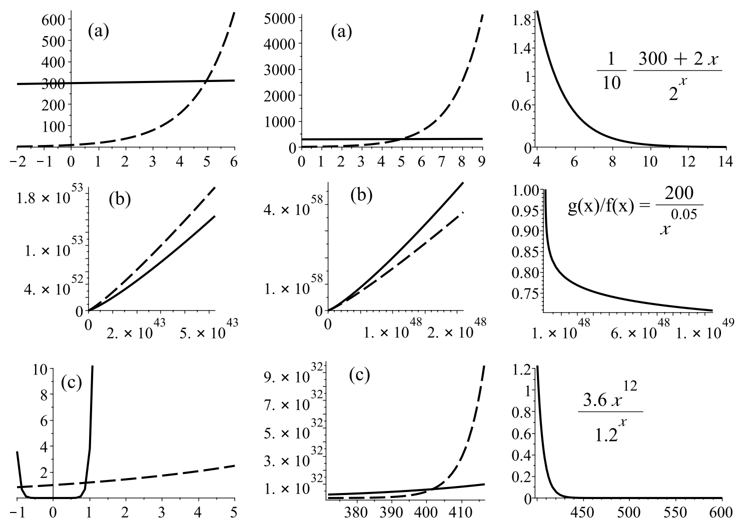
14) $R(x) = \dfrac{(2\,x+3) \cdot (x+1)}{(3\,x+2) \cdot (3\,x-2)}$ has y-intercept $R(0) = \dfrac{-3}{4}$, x-intercepts -1 and $\dfrac{-3}{2}$,
vertical asymptotes $x = \dfrac{-2}{3}$ and $x = \dfrac{2}{3}$, horizontal asymptote $y = \dfrac{2}{9}$ because $\lim_{x \to \pm\infty} R(x) = \lim_{x \to \pm\infty} \dfrac{2\,x^2}{9\,x^2} = \dfrac{2}{9}$

15) In each part, function f is graphed with a solid curve and function g is graphed with a dashed curve for the lefthand and middle figures; the righthand figure plots a labeled ratio for $\dfrac{\text{eventually smaller}}{\text{eventually larger}}$. (Note: some plots do not include the origin.)

a) $f(x) = 300 + 2\,x$ and $g(x) = 10 \cdot (2^x)$,
$f(x) = g(x)$ at $x \approx 4.95$;
$g(x)$ dominates as $x \to \infty$.

b) $f(x) = 5\,x^{1.2}$ and $g(x) = 1000\,x^{1.15}$,
$f(x) = g(x)$ if $x = 200^{20} \approx 1.05 \times 10^{46}$;
$f(x)$ dominates as $x \to \infty$ because
$$\lim_{x \to \infty} \frac{g(x)}{f(x)} = \lim_{x \to \infty} \frac{1000\,x^{1.15}}{5\,x^{1.2}} = \lim_{x \to \infty} \frac{200}{x^{0.05}} = 0.$$

c) $f(x) = 3.6\,x^{12}$ and $g(x) = 1.2^x$;
$f(x) = g(x)$ at $x \approx -0.89, 0.91, 401.6$;
$g(x)$ dominates as $x \to \infty$.



16) Starting with 50 grams and with a 13-hour half-life, the mass remaining after t hours is $m(t) = 50 \cdot \left(\frac{1}{2}\right)^{t/13}$.

Thus $m(26) = 50 \cdot \left(\frac{1}{2}\right)^2 = 12.5$ grams. The mass will be 10 grams after $t = 13 \cdot \dfrac{\log(10/50)}{\log(1/2)} \approx 30.185$ hours.

17) A trip at constant speed S miles-per-hour for time T hours will travel distance equal to $S \cdot T$ miles. Since distance between Long Island and Albany is constant, travel time T and speed S of trip are inversely proportional to each other , i.e.,

$$T = \frac{\text{distance}}{S} \qquad \text{and} \qquad S = \frac{\text{distance}}{T}$$

If a trip at 55 miles-per-hour takes 3.5 hours, then the distance is $(55) \cdot (3.5) = 192.5$ miles. In order to complete the trip in 3 hours, the speed will need to be $S = \dfrac{192.5}{3} \approx 64.17$ miles-per-hour.

18) The linear function through $(4, 70)$ and $(9, 157.5)$, $L(x) = 70 + \dfrac{157.5 - 70}{9 - 4} \cdot (x - 4) = 17.5\,x$, turns out to be a power function. Base b of exponential function E thru those points satisfies equation $b^5 = \dfrac{a\,b^9}{a\,b^4} = \dfrac{157.5}{70}$, thus $E(x) = 70 \cdot \left(\dfrac{157.5}{70}\right)^{(x-4)/5}$

19)

part	$f(x)$	$a\,b^t$	a	b	$r = b - 1$
a)	$30\,e^{0.2\,x}$	$30\,(1.22140^t)$	30	$e^{0.2}$	$22.1403\,\%$
b)	$75\,e^{-0.08\,x}$	$75\,(0.92312^t)$	75	$e^{-0.08}$	$-7.6884\,\%$
c)	$36\,(1.2^{x-1})$	$30\,(1.2^t)$	$36/1.2 = 30$	1.2	$20\,\%$
d)	$36\,(0.8^{-x+1})$	$28.8\,(1.25^t)$	$36 \times 0.8 = 28.8$	$0.8^{-1} = 1.25$	$25\,\%$
e)	$36\,(1.2^{2\,x})$	$36\,(1.44^t)$	36	$1.2^2 = 1.44$	$44\,\%$
f)	$36\,(0.6^x)$	$36\,(0.6^t)$	36	0.6	$-40\,\%$

20)

part	$f(x)$	$k\,x^p$	k	p
a)	$30\sqrt{4x}$	$60\,x^{1/2}$	$30\cdot\sqrt{4}=60$	$1/2$
b)	$\dfrac{75}{3x^2}$	$25\,x^{-2}$	$\dfrac{75}{3} = 25$	-2
c)	$5\sqrt[3]{8x^2}$	$10\,x^{2/3}$	$5\cdot\sqrt[3]{8}=10$	$2/3$
d)	$\dfrac{75}{\sqrt[4]{81x^3}}$	$25\,x^{-3/4}$	$\dfrac{75}{\sqrt[4]{81}} = 25$	$\dfrac{-3}{4}$

21)

part	$f(x)$	linear	quadratic	exponential	power	polynomial	rational	neither
a)	$5\,x$	L			Pwr	Pol	R	
b)	$\frac{1}{3}x^2$		Q		Pwr	Pol	R	
c)	$5x + \frac{1}{3}x^2$		Q			Pol	R	
d)	$5\sqrt{x} + \frac{1}{3}x^2$							N
e)	$\frac{3}{x^2}$				Pwr		R	
f)	$5 + \frac{3}{x^2}$						R	
g)	$3.4\,x^{1.2}$				Pwr			
h)	$3.4\,(1.2^x)$			E				

Note: although (d) and (f) are sums of power functions, neither is a polynomial because exponents 1/2 and −2 are not non-negative integers.

22) An angle of one degree is $1^\circ \times \dfrac{\pi}{180^\circ} \approx 0.017\,453$ radians. On a circle (or sphere) with radius 3960 miles, the corresponding arc has length $r\cdot\theta = 3960\cdot\dfrac{\pi}{180} \approx 69.115\,038 \approx 69$ miles. [Using *nautical miles* as distance unit would simplify this answer.]

23) Let c be the distance between points A and B; apply Law of Cosines.

$$c^2 = a^2 + b^2 - 2\,a\,b\,\cos(\angle C) = 3.5^2 + 2.8^2 - 2\cdot(3.5)\cdot(2.8)\cdot\cos(42.3^\circ) \approx 5.593\,231 \approx 2.3650^2$$

The distance "across the lake" is about 2.4 miles.

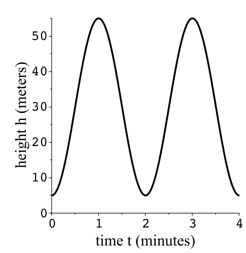
24) Angles within the ABC triangle are $\angle A = \angle CAB = 32^\circ$, $\angle B = \angle CBA = 48^\circ$, and $\angle C = \angle ACB = 180^\circ - (\angle A + \angle B) = 100^\circ$. The known distance, 5 miles between A and B, is opposite $\angle C$. The Law of Sines statement $\dfrac{CA}{\sin(48^\circ)} = \dfrac{5}{\sin(100^\circ)}$ implies the plane's distance from A is $CA = \dfrac{\sin(48^\circ)}{\sin(100^\circ)} \times 5 \approx 3.773$ miles. Therefore, the plane's height above ground is

$$(CA)\times\sin(\angle A) \;=\; \frac{\sin(48^\circ)\times\sin(32^\circ)}{\sin(100^\circ)}\times 5 \;\approx\; 1.999\,410 \;\approx\; 2.00 \text{ miles}$$

25) If θ ends in quadrant II, then $\cos(\theta) < 0$. Therefore $\cos(\theta) = -\sqrt{1-\sin(\theta)^2} = -\sqrt{1-0.6^2} = -0.8$. That implies $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)} = \dfrac{0.6}{-0.8} = -0.75$, $\sin(2\,\theta) = 2\,\sin(\theta)\,\cos(\theta) = 2\cdot(0.6)\cdot(-0.8) = -0.96$, and

$$\cos\left(\theta + \frac{\pi}{4}\right) = \cos(\theta)\cdot\cos\left(\frac{\pi}{4}\right) - \sin(\theta)\cdot\sin\left(\frac{\pi}{4}\right) = (-0.8)\cdot\frac{\sqrt{2}}{2} - (0.6)\cdot\frac{\sqrt{2}}{2} = (-1.4)\cdot\frac{\sqrt{2}}{2} = (-0.7)\cdot\sqrt{2}$$

26) The height function has amplitude 25 meters (radius of the wheel), midline $h = 30$ meters (height of wheel's center = minimum + amplitude), and period 2 minutes (time for one revolution). Angular velocity is constant, half of each period (one minute per revolution) is spent at or above the midline, 30 meters.
Since clock starts ($t = 0$) when boarding at lowest point ($5 = 30 - 25$), the height function can be written as $h(t) = 30 - 25\,\cos(\pi\,t)$.



27) $0 = \cos(x) + 2\,\cos(x)^2 = \cos(x)\cdot\Big(1 + 2\cos(x)\Big)$ is equivalent to $0 = \cos(x)$ or $0 = 1 + 2\cos(x)$. The solutions in $[0, 2\pi]$ are

- $x_1 = \arccos(0) = \pi/2$,
- $x_2 = x_1 + \pi = 3\pi/2$,
- $x_3 = \arccos(-1/2) = 2\pi/3$,
- $x_4 = 2\pi - x_3 = 4\pi/3$.

28) Lefthand figure shows the Cartesian graph of $y = 8 - 12\ \cos(x)$ and the righthand figure shows the Polar graph of $r = 8 - 12\ \cos(\theta)$.